STRONG CONVERGENCE FOR THE MODIFIED MANN'S ITERATION OF λ-STRICT PSEUDOCONTRACTION

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Abstract. In this paper, for an λ-strict pseudocontraction T, we prove strong convergence of the modified Mann's iteration defined by

$$x_{n+1} = \beta_n u + \gamma_n x_n + (1 - \beta_n - \gamma_n)[\alpha_n T x_n + (1 - \alpha_n)x_n],$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0, 1)$ satisfy:

(i) $0 \le \alpha_n \le \frac{\lambda}{K^2}$ with $\liminf\limits_{n\to\infty} \alpha_n(\lambda - K^2\alpha_n) > 0$;

(ii) $\lim\limits_{n\to\infty} \beta_n = 0$ and $\sum\limits_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\limsup\limits_{n\to\infty} \gamma_n < 1$.

Our results unify and improve some existing results.

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Throughout this paper, let E be a Banach space with the norm $\|\cdot\|$ and the dual space E^* and $\langle y, x^*\rangle$ denote the value of $x^* \in E^*$ at $y \in E$. The normalized duality mapping J from E into 2^{E^*} is defined by the following equation:

$$J(x) = \{x^* \in E^* : \langle x, x^*\rangle = \|x\|\|x^*\|, \|x\| = \|x^*\|\}.$$

Let $F(T) = \{x \in E : Tx = x\}$, the set of all fixed point of a mapping T.

Recall that a mapping T with domain $D(T)$ and range $R(T)$ in Banach space E is called *Lipschitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \le L\|x - y\| \text{ for all } x, y \in D(T).$$

T is said to be *nonexpansive* if $L = 1$ in the above inequality. T is called λ-*strictly pseudocontractive* if there exists $\lambda \in (0, 1)$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \le ||x - y||^2 - \lambda||x - y - (Tx - Ty)||^2 \text{ for all } x, y \in D(T). \quad (1)$$

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T is called *pseudocontractive* if $\lambda \equiv 0$ in (1). Obviously, each λ-strictly pseudocontractive mapping is a Lipschitzian and pseudocontractive mapping with $L = \frac{\lambda+1}{\lambda}$. In particular, a nonexpansive mapping is λ-strictly pseudocontractive mapping in a Hilbert space, but the conversion may be false.

For finding a fixed point of λ-strictly pseudocontractive mapping T, a strong convergence theorem was obtained by Zhou [12] in a 2-uniformly smooth Banach space.

Theorem Z. (Zhou [12, Theorem 2.3]) Let C be a closed convex subset of a real 2-uniformly smooth Banach space E and let $T : C \to C$ be a λ-strict pseudo-contraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by

$$x_{n+1} = \beta_n u + \gamma_n x_n + (1 - \beta_n - \gamma_n)[\alpha_n T x_n + (1 - \alpha_n)x_n], \tag{2}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0, 1)$ satisfy:

(i) $\alpha_n \in [a, \mu]$, $\mu = \min\{1, \frac{\lambda}{K^2}\}$ for some constant $a \in (0, \mu)$;

(ii) $\lim\limits_{n\to\infty} \beta_n = 0$ and $\sum\limits_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\lim\limits_{n\to\infty} |\alpha_{n+1} - \alpha_n| = 0$;

(iv) $0 < \liminf\limits_{n\to\infty} \gamma_n \le \limsup\limits_{n\to\infty} \gamma_n < 1$.

Then the sequence $\{x_n\}$ converges strongly to a fixed point z of T.

Recently, Zhang and Su [13] extended Zhou's results to q-uniformly smooth Banach space. We also note that the above results excluded $\gamma_n \equiv 0$ and $\gamma_n = \frac{1}{n+1}$. Very recently, Chai and Song [1] studied the strong convergence of the modified Mann's iteration (2) with $\gamma_n \equiv 0$.

Theorem CS. (Chai and Song [1, Theorem 3.1]) Let C be a closed convex subset of a real 2-uniformly smooth Banach space E and let $T : C \to C$ be a λ-strict pseudo-contraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by

$$x_{n+1} = \beta_n u + (1 - \beta_n)[\alpha_n T x_n + (1 - \alpha_n)x_n], \tag{3}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ in $(0, 1)$ satisfy the following control conditions:

(i) $\alpha_n \in [a, \mu]$, $\mu = \min\{1, \frac{\lambda}{K^2}\}$ for some constant $a \in (0, \mu)$;

(ii) $\sum\limits_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$;

(iii) $\lim\limits_{n\to\infty} \beta_n = 0$, $\sum\limits_{n=1}^{\infty} \beta_n = \infty$ and $\sum\limits_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$.

Then, the sequence $\{x_n\}$ converges strongly to a fixed point z of T.

In this paper, we will deal with strong convergence of the modified Mann's iteration (2) under more relaxed conditions on the sequences $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0, 1)$,

(i) $\alpha_n \in [0, \mu]$, $\mu = \min\{1, \frac{\lambda}{K^2}\}$ with $\liminf\limits_{n\to\infty} \alpha_n(\lambda - K^2\alpha_n) > 0$;

(ii) $\lim\limits_{n\to\infty} \beta_n = 0$ and $\sum\limits_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\limsup\limits_{n\to\infty} \gamma_n < 1$.

Our results obviously develop and complement the corresponding ones of Zhou [12], Song and Chai [9], Chai and Song [1], Zhang and Su [13] and others. Moreover,

our conditions are simpler, which contain $\gamma_n \equiv 0$ and $\gamma_n = \frac{1}{n+1}$ as special cases. Our conclusions may be regarded as a unification of the some existing results.

For achieving our purposes, the following facts and results are needed. Let $\rho_E : [0,\infty) \to [0,\infty)$ be the modulus of smoothness of E defined by

$$\rho_E(t) = \sup\left\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : x \in S(E), \|y\| \leq t\right\}.$$

Let $q > 1$. A Banach space E is said to be *q-uniformly smooth* if there exists a fixed constant $c > 0$ such that $\rho_E(t) \leq ct^q$ and *uniformly smooth* if $\lim_{t\to 0} \frac{\rho_E(t)}{t} = 0$. Clearly, a q-uniformly smooth space must be uniformly smooth. Typical example of uniformly smooth Banach spaces is L_p $(p > 1)$. More precisely, L_p is $\min\{p, 2\}$-uniformly smooth for every $p > 1$.

Lemma 1. (Zhou [12, Lemma 1.2]) *Let C be a nonempty subset of a real 2-uniformly smooth Banach space E with the best smooth constant K, and let $T : C \to C$ be a λ-strict pseudocontraction. For any $\alpha \in (0,1)$, we define $T_\alpha = (1-\alpha)x + \alpha Tx$. Then,*

$$\|T_\alpha x - T_\alpha y\|^2 \leq \|x-y\|^2 - 2\alpha(\lambda - K^2\alpha)\|Tx - Ty - (x-y)\|^2 \text{ for all } x, y \in C. \quad (4)$$

In particular, as $\alpha \in (0, \frac{\lambda}{K^2}]$, $T_\alpha : C \to C$ is nonexpansive such that $F(T_\alpha) = F(T)$.

Lemma 2 was shown and used by several authors. For detail proofs, see Liu [2] and Xu [10, 11]. Furthermore, a variant of Lemma 1 has already been used by Reich in [6, Theorem 1].

Lemma 2. *Let $\{a_n\}$ be a sequence of nonnegative real numbers such that*

$$a_{n+1} \leq (1-t_n)a_n + t_n c_n, \quad \forall\ n \geq 0.$$

Assume that $\{t_n\} \subset [0,1]$ and $\{c_n\} \subset (0,+\infty)$ satisfy the restrictions:

$$\sum_{n=0}^{\infty} t_n = \infty \text{ and } \limsup_{n\to\infty} c_n \leq 0.$$

Then as $n \to \infty$, $\{a_n\}$ converges to zero.

Morales and Jung [3], in 2000, proved the following behavior for pseudocontractive mappings. Also see Song and Chen [7, 8] for more details. The same result of nonexpansive mapping was shown by Reich [5] in 1980.

Lemma 3. ([3, 7, 8]) *Let C be a nonempty, closed and convex subset of a uniformly smooth Banach space E, and let $T : C \to C$ be a continuous pseudocontractive mapping with $F(T) \neq \emptyset$. Suppose that for $t \in (0,1)$ and $u \in C$, x_t defined by*

$$x_t = tu + (1-t)Tx_t. \quad (5)$$

Then, as $t \to 0$, x_t converges strongly to a fixed point of T.

This following results play a key role in proving our main results, which was proved by Song and Chen [7].

Lemma 4. (Song and Chen [7, Theorm 2.3]) *Let C be a nonempty, closed and convex subset of a uniformly smooth Banach space E, and let $T : C \to C$ be a continuous pseudocontractive mapping with a fixed point. Assume that there exists a bounded sequence $\{x_n\}$ such that $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ and $z = \lim_{t\to 0} z_t$ exists, where $\{z_t\}$ is defined by (5). Then*

$$\limsup_{n\to\infty} \langle u - z, J(x_n - z)\rangle \le 0.$$

We also need the following results that showed by Mainge in 2008.

Lemma 5. (Mainge [4, Lemma 3.1]) *Let $\{\Gamma_n\}$ be a sequence of real numbers that does not decrease at infinity, in the sense that there exists a subsequence $\{\Gamma_{n_k}\}$ of $\{\Gamma_n\}$ such that*

$$\Gamma_{n_k} < \Gamma_{n_k+1} \text{ for all } k \ge 0.$$

Also consider the sequence of integers $\{\tau(n)\}_{n\ge n_0}$ defined by

$$\tau(n) = \max\{k \le n; \Gamma_k < \Gamma_{k+1}\}.$$

Then $\tau(n)$ is a nondecreasing sequence verifying

$$\lim_{n\to\infty} \tau(n) = +\infty,$$

and, for all $n \ge n_0$, the following two estimates hold:

$$\Gamma_{\tau(n)} \le \Gamma_{\tau(n)+1} \text{ and } \Gamma_n \le \Gamma_{\tau(n)+1}.$$

Next we will show our main results.

Theorem 6. *Let C be a closed convex subset of a real 2-uniformly smooth Banach space E and let $T : C \to C$ be a λ-strict pseudo-contraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by the modified Mann's iteration (2), where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0, 1)$ satisfy:*

(i) $\alpha_n \in [0, \mu]$, $\mu = \min\{1, \frac{\lambda}{K^2}\}$ with $\liminf_{n\to\infty} \alpha_n(\lambda - K^2\alpha_n) > 0$;

(ii) $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\limsup_{n\to\infty} \gamma_n < 1$.

Then the sequence $\{x_n\}$ converges strongly to a fixed point z of T.

Proof. Let $y_n = T_{\alpha_n} x_n = \alpha_n T x_n + (1-\alpha_n)x_n$. Then for each n, T_{α_n} is nonexpansive and $F(T) = F(T_{\alpha_n})$ by Lemma 1. So, the sequence $\{x_n\}$ is bounded since for given $p \in F(T) = F(T_{\alpha_n})$,

$$\begin{aligned}
\|x_{n+1} - p\| &= \|\beta_n(u - p) + \gamma_n(x_n - p) + (1 - \beta_n - \gamma_n)(T_{\alpha_n} x_n - p)\| \\
&\le \beta_n\|u - p\| + \gamma_n\|x_n - p\| + (1 - \beta_n - \gamma_n)\|T_{\alpha_n} x_n - T_{\alpha_n} p\| \\
&\le \beta_n\|u - p\| + \gamma_n\|x_n - p\| + (1 - \beta_n - \gamma_n)\|x_n - p\| \\
&\le \beta_n\|u - p\| + (1 - \beta_n)\|x_n - p\| \\
&\le \max\{\|x_n - p\|, \|u - p\|\} \\
&\ \ \vdots \\
&\le \max\{\|x_0 - p\|, \|u - p\|\}.
\end{aligned}$$

Now we show $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. It follows from Lemma 1 that

$$\|y_n - p\| = \|T_{\alpha_n} x_n - p\|^2 \le \|x_n - p\|^2 - 2\alpha_n(\lambda - K^2\alpha_n)\|x_n - Tx_n\|^2. \tag{6}$$

Furthermore, we also have

$$\begin{aligned}
&\|x_{n+1}-p\|^2 = \|\beta_n(u-p)+\gamma_n(x_n-p)+(1-\beta_n-\gamma_n)(y_n-p)\|^2\\
\leq&\beta_n\|u-p\|^2+\gamma_n\|x_n-p\|^2\\
&+(1-\beta_n-\gamma_n)(\|x_n-p\|^2-2\alpha_n(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2)\\
\leq&\beta_n\|u-p\|^2+(1-\beta_n)\|x_n-p\|^2\\
&-2\alpha_n(1-\beta_n-\gamma_n)(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2\\
\leq&\|x_n-p\|^2-(2\alpha_n(1-\beta_n-\gamma_n)(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2-\beta_n\|u-p\|^2).
\end{aligned}$$

Then we obtain

$$2\alpha_n(1-\beta_n-\gamma_n)(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2 \leq \|x_n-p\|^2-\|x_{n+1}-p\|^2+\beta_n\|u-p\|^2.$$

It follows from Lemma 3 that there exist $z \in F(T)$ and $x_t = tu+(1-t)Tx_t$ such that $\lim_{t\to 0} x_t = z$. Then we also have

$$2\alpha_n(1-\beta_n-\gamma_n)(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2 \leq \|x_n-z\|^2-\|x_{n+1}-z\|^2+\beta_n\|u-z\|^2. \quad (7)$$

Following the proof technique in Mainge [4, Lemma 3.2, Theorem 3.1], the proof may be divided two cases.

Case 1. If there exists N_0 such that the sequence $\{\|x_n-z\|^2\}$ is nonincreasing for $n \geq N_0$, then the limit $\lim_{n\to\infty}\|x_n-z\|^2$ exists, and hence $\lim_{n\to\infty}(\|x_n-z\|^2-\|x_{n+1}-z\|^2)=0$. So by the condition (ii) and the inquality (7), it is obvious that

$$\limsup_{n\to\infty}\alpha_n(1-\beta_n-\gamma_n)(\lambda-K^2\alpha_n)\|x_n-Tx_n\|^2=0.$$

It follows from the conditions (i), (ii) and (iii) that

$$\lim_{n\to\infty}\|x_n-Tx_n\|=0. \quad (8)$$

Then by Lemma 4, we obtain

$$\limsup_{n\to\infty}\langle u-z, J(x_{n+1}-z)\rangle \leq 0. \quad (9)$$

Finally, we show that $x_n \to z$. Indeed, since

$$\begin{aligned}
\|x_{n+1}-z\|^2 =&\langle(\beta_n u+\gamma_n x_n+(1-\beta_n-\gamma_n)T_{\alpha_n}x_n)-z, J(x_{n+1}-z)\rangle\\
\leq&\beta_n\langle u-z, J(x_{n+1}-z)\rangle+\gamma_n\|x_n-z\|\|J(x_{n+1}-z)\|\\
&+(1-\beta_n-\gamma_n)\|T_{\alpha_n}x_n-z\|\|J(x_{n+1}-z)\|\\
\leq&\beta_n\langle u-z, J(x_{n+1}-z)\rangle+(1-\beta_n)\|x_n-z\|\|x_{n+1}-z\|\\
\leq&\beta_n\langle u-z, J(x_{n+1}-z)\rangle+(1-\beta_n)\frac{\|x_n-z\|^2+\|x_{n+1}-z\|^2}{2},
\end{aligned}$$

then, we have

$$\|x_{n+1}-z\|^2 \leq (1-\beta_n)\|x_n-z\|^2+2\beta_n\langle u-z, J(x_{n+1}-z)\rangle. \quad (10)$$

So, an application of Lemma 2 onto (10) yields that $\lim_{n\to\infty}\|x_n-z\|=0$.

Case 2. Assume that there exists a subsequence $\{\|x_{n_k}-z\|^2\}$ of $\{\|x_n-z\|^2\}$ such that $\|x_{n_k}-z\|^2 < \|x_{n_k+1}-z\|^2$ for for all $k \geq 0$. Let

$$\Gamma_n = \|x_n-z\|^2 \text{ and } \tau(n)=\max\{k\leq n; \Gamma_k<\Gamma_{k+1}\}.$$

It follows from Lemma 5 that $\tau(n)$ is a nondecreasing sequence verifying

$$\lim_{n\to\infty} \tau(n) = +\infty$$

and for n large enough,

$$\Gamma_{\tau(n)} \le \Gamma_{\tau(n)+1},\ \Gamma_n = \|x_n - z\|^2 \le \Gamma_{\tau(n)+1}. \tag{11}$$

In light of Eq. (7), we have

$$2\alpha_{\tau(n)}(1-\beta_{\tau(n)}-\gamma_{\tau(n)})(\lambda - K^2\alpha_{\tau(n)})\|x_{\tau(n)} - Tx_{\tau(n)}\|^2 \le \beta_{\tau(n)}\|u-z\|^2,$$

and so by the condition (i),(ii) and (iii), we have

$$\lim_{n\to\infty} \|x_{\tau(n)} - Tx_{\tau(n)}\| = 0.$$

Then as $n \to \infty$,

$$\begin{aligned}\|x_{\tau(n)+1} - Tx_{\tau(n)}\| \le& \beta_{\tau(n)}\|u - Tx_{\tau(n)}\| + \gamma_{\tau(n)}\|x_{\tau(n)} - Tx_{\tau(n)}\| \\ &+ (1-\beta_{\tau(n)}-\gamma_{\tau(n)})(1-\alpha_{\tau(n)})\|x_{\tau(n)} - Tx_{\tau(n)}\| \to 0.\end{aligned}$$

Since

$$\begin{aligned}\|x_{\tau(n)+1} - Tx_{\tau(n)+1}\| &\le \|x_{\tau(n)+1} - Tx_{\tau(n)}\| + \|Tx_{\tau(n)} - Tx_{\tau(n)+1}\| \\ &\le \|x_{\tau(n)+1} - Tx_{\tau(n)}\| + \|x_{\tau(n)} - x_{\tau(n)+1}\| \\ &\le 2\|x_{\tau(n)+1} - Tx_{\tau(n)}\| + \|x_{\tau(n)} - Tx_{\tau(n)}\|,\end{aligned}$$

we have

$$\lim_{n\to\infty} \|x_{\tau(n)+1} - Tx_{\tau(n)+1}\| = 0. \tag{12}$$

Then by Lemma 4, we obtain

$$\limsup_{n\to\infty} \langle u - z, J(x_{\tau(n)+1} - z)\rangle \le 0. \tag{13}$$

Using the similar proof techniques of Case 1, the only modification is that n is replaced by $\tau(n)$, we have

$$\|x_{\tau(n)+1} - z\|^2 \le (1-\beta_{\tau(n)})\|x_{\tau(n)} - z\|^2 + 2\beta_{\tau(n)}\langle u - z, J(x_{\tau(n)+1} - z)\rangle. \tag{14}$$

Together with (11), we have

$$\Gamma_{\tau(n)} \le \Gamma_{\tau(n)+1} \le (1-\beta_{\tau(n)})\Gamma_{\tau(n)} + 2\beta_{\tau(n)}\langle u - z, J(x_{\tau(n)+1} - z)\rangle,$$

and so,

$$\Gamma_{\tau(n)} = \|x_{\tau(n)} - z\|^2 \le 2\langle u - z, J(x_{\tau(n)+1} - z)\rangle.$$

Along with (13), we have

$$\lim_{n\to\infty} \Gamma_{\tau(n)} = \lim_{n\to\infty} \|x_{\tau(n)} - z\| = 0.$$

It follows from (14), (13) and the condition (ii) that

$$\lim_{n\to\infty} \Gamma_{\tau(n)+1} = \lim_{n\to\infty} \|x_{\tau(n)+1} - z\| = 0.$$

Now it follows from (11) that

$$\lim_{n\to\infty} \Gamma_n = \lim_{n\to\infty} \|x_n - z\| = 0.$$

The proof is completed. □

Clearly, Theorem 6 contains $\gamma_n \equiv 0$ and $\gamma_n = \frac{1}{n+1}$ as special cases. So the following result is obtained easily.

Corollary 7. *Let C be a closed convex subset of a real 2-uniformly smooth Banach space E and let $T : C \to C$ be a λ-strict pseudo-contraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by the modified Mann's iteration (3), where $\{\alpha_n\}$ and $\{\beta_n\}$ in $(0,1)$ satisfy:*

(i) $\alpha_n \in [0,\mu]$, $\mu = \min\{1, \frac{\lambda}{K^2}\}$ with $\liminf_{n\to\infty} \alpha_n(\lambda - K^2\alpha_n) > 0$;

(ii) $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \beta_n = \infty$.

Then the sequence $\{x_n\}$ converges strongly to a fixed point z of T.

Using the same proof techniques as Theorem 6, we easily obtain the following result. Since the only difference is that $\alpha_n(\lambda - K^2\alpha_n)$ is replaced by $\alpha_n(q\lambda - C_q\alpha_n^{q-1})$ in its proof, so we omit its proof.

Theorem 8. *Let C be a closed convex subset of a real q-uniformly smooth Banach space E $(q > 1)$ and let $T : C \to C$ be a λ-strict pseudo-contraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by the modified Mann's iteration (2) or (3), where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0,1)$ satisfy:*

(i) $\alpha_n \in [0,\mu]$, $\mu = \min\{1, \{\frac{q\lambda}{C_q}\}^{\frac{1}{q-1}}\}$ with $\liminf_{n\to\infty} \alpha_n(q\lambda - C_q\alpha_n^{q-1}) > 0$;

(ii) $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\limsup_{n\to\infty} \gamma_n < 1$.

Then the sequence $\{x_n\}$ converges strongly to a fixed point z of T.

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